The EPR Paradox
and
Bell's Inequality and Entanglement
by D. Petcher

Background

▶ Einstein did not like quantum theory (despite the fact that he had a lot to do with its development)
  ▶ uncertainty principle
  ▶ unpredictability
  ▶ he thought it was “incomplete”
▶ “God does not play dice with the universe.” — Einstein
▶ “Quit telling God what to do!” — Niels Bohr

Background

▶ In 1935, Einstein, Boris Podolsky, and Nathan Rosen (EPR) proposed a thought experiment demonstrating the incompleteness of quantum theory. Assumptions for completeness:
  ▶ a particle should have a separate reality, independent of measurement
  ▶ all properties of the particle should exist, in principle, at any given time

Background

▶ They proposed this thought experiment: a spinless particle decays into two particles with “spin” (a quantum property of some particles, which behaves like a spinning charge)

▶ once they separate, the result of measurements on either spin should not affect the state of the other
Spin in Inhomogeneous Magnetic Field

Measuring Spin

Classical and Quantum Behavior - electron (spin $\frac{1}{2}$)

Classical - a range of scatter depending on the angle
Quantum - "up" or "down" (relative to the magnet)

EPR Thought Experiment

- zero spin particle decays into spin up and spin down
- total spin is still zero $\iff$ spin is conserved
EPR Thought Experiment

- Spins are deflected, up or down.
- If the magnetic fields are lined up, the results are correlated — when one goes up, the other goes down.

The Paradox

- In quantum theory, measuring spin on the x-axis and measuring spin on the y-axis (i.e., two perpendicular axes) is rather like measuring position and momentum. You cannot measure both precisely. (Uncertainty principle)
- But if we measure one spin along the x-axis, we know in principle that the other must be of opposite spin along the x-axis.
- Therefore, we just measure the spin of the second along the y-axis and we know both precisely!

\Rightarrow \text{something must be wrong with quantum mechanics}

Bell’s Inequality

- In 1964, John Bell derived an inequality using a weaker assumption than that of EPR, just that after the particles separate, measuring a particle on one side does not affect the measurement of the particle on the other.
- The inequality therefore represents a statement that follows from a “complete” theory, as Einstein put it.
- Quantum mechanics predicts a violation of the inequality.
- Therefore the issue could be decided experimentally:
  - Does experiment obey or violate the inequality?

Example: rotating one magnet by 20°

- Note “up” and “down” for rotated magnet are different.
Sample Mechanism for Predetermined Outcomes

So rotating the magnet would make a change in some measurements.

Another way to look at it...

Set theory and Venn diagrams:
- If have three categories: i.e. Male/Female, Tall/Short, Brown Eyes/Blue Eyes
- Let set A = males; set B = tall; set C = blue eyes
- Can show:
  - \( P(A, \text{not } B) + P(B, \text{not } C) \geq P(A, \text{not } C) \)
  - \( P(\text{male, not tall}) + P(\text{tall, not blue}) \geq P(\text{male, not blue}) \)

Another way to look at it...

Rotate one or both magnets

First sequence on left:

\[
\uparrow\downarrow \quad \uparrow\downarrow \quad \uparrow\downarrow \quad \uparrow\downarrow \quad \uparrow\downarrow \quad \ldots
\]

Perfectly correlated sequence on right:

\[
\downarrow\uparrow \quad \uparrow\downarrow \quad \uparrow\downarrow \quad \uparrow\downarrow \quad \downarrow\uparrow \quad \downarrow\uparrow \quad \ldots
\]

Rotate magnetic field on right by angle 20°:

\[
\downarrow\uparrow \quad \downarrow\uparrow \quad \uparrow\downarrow \quad \uparrow\downarrow \quad \uparrow\downarrow \quad \uparrow\downarrow \quad \ldots
\]

Rotate magnetic field on left by angle −20°:

\[
\uparrow\downarrow \quad \uparrow\downarrow \quad \downarrow\uparrow \quad \downarrow\uparrow \quad \downarrow\uparrow \quad \downarrow\uparrow \quad \ldots
\]
Another way to look at it...

Now equate sets with experimental outcomes:

<table>
<thead>
<tr>
<th>Set</th>
<th>Example</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>male</td>
<td>at -20°: up on left ⇔ down on right</td>
</tr>
<tr>
<td>B</td>
<td>tall</td>
<td>at 0°: up on left ⇔ down on right</td>
</tr>
<tr>
<td>C</td>
<td>blue eyes</td>
<td>at 20°: up on left ⇔ down on right</td>
</tr>
</tbody>
</table>

⇒ P(A, not B) + P(B, not C) ≥ P(A, not C)

The argument

- all we need do is classify states into categories, of whether they are up on left down on right for given angles
- this is sufficient to derive the inequality
- then we check it with quantum theory
- finally we check it with experiment

Rotate by 20° and -20°

<table>
<thead>
<tr>
<th>Number of Each Type</th>
<th>Left Observer</th>
<th>Right Observer</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-20° 0° 20°</td>
<td>-20° 0° 20°</td>
</tr>
<tr>
<td>N₁</td>
<td>↑ ↑ ↑</td>
<td>↓ ↓ ↓</td>
</tr>
<tr>
<td>N₂</td>
<td>↑ ↑ ↓</td>
<td>↓ ↓ ↑</td>
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<tr>
<td>N₃</td>
<td>↑ ↓ ↑</td>
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<td>↑ ↑ ↓</td>
</tr>
<tr>
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Probability

- the probability for getting any particular outcome of particle states is the same as the fraction of those states relative to the whole population:
  - \( N_{\text{total}} = N₁ + N₂ + N₃ + N₄ + N₅ + N₆ + N₇ + N₈ \)
  - \( P₁ = N₁ / N_{\text{total}}, P₂ = N₂ / N_{\text{total}}, \text{etc.} \)
  - \( P₁ + P₂ + \cdots + P₈ = 1 \)
Bell’s Inequality

- Consider the probability of both sides registering an “up” value for spin, relative to the particular axis of the magnetic field. (anti-correlated cases)
- Define \( P(\theta_1, \theta_2) \) to mean “the probability that measuring on the left with the angle of the magnet set to \( \theta_1 \) gives “up” and also measuring on the right with the angle of the magnet set to \( \theta_2 \) gives “up”.
- We wish to show \( P(-20^\circ, 0^\circ) + P(0^\circ, 20^\circ) \geq P(-20^\circ, 20^\circ) \)

\[
P(-20^\circ, 0^\circ) = \left( \frac{N_3 + N_4}{N_{total}} \right) = P_3 + P_4
\]

\[
P(0^\circ, 20^\circ) = \left( \frac{N_2 + N_6}{N_{total}} \right) = P_2 + P_6
\]

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<tr>
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<td>↓ ↑ ↑</td>
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<tr>
<td>( N_4 )</td>
<td>↓ ↓ ↑</td>
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<tr>
<td>( N_5 )</td>
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<tr>
<td>( N_7 )</td>
<td>↓ ↓ ↑</td>
<td>↑ ↑ ↓</td>
</tr>
<tr>
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Bell's Inequality

- $P(-20^\circ, 20^\circ) = P_2 + P_4$
- $P(-20^\circ, 0^\circ) = P_3 + P_4$
- $P(0^\circ, 20^\circ) = P_2 + P_6$
- so $P(-20^\circ, 0^\circ) + P(0^\circ, 20^\circ) \geq P(-20^\circ, 20^\circ)$
  is just $P_3 + P_4 + P_2 + P_6 \geq P_2 + P_4$
- the result can be generalized to arbitrary angles:
  $\Rightarrow P(-\theta, 0^\circ) + P(0^\circ, \theta) \geq P(-\theta, \theta)$

Check with $\theta = 20^\circ$

- $\frac{1}{2} \sin^2(10^\circ) + \frac{1}{2} \sin^2(10^\circ) \geq \frac{1}{2} \sin^2(20^\circ)$ ???
- $\frac{1}{2} \sin^2(20^\circ) = 0.058$
- $\frac{1}{2} \sin^2(10^\circ) = 0.015$
- We would need $0.015 + 0.015 = 0.030 \geq 0.058$
  but not true!!!
- so quantum mechanics violates Bell's inequality
- which is correct? classical? quantum? $\Rightarrow$ a question for experiment

Summary

- if the measurement on one side does not affect the measurement on the other (each is already set once the decay occurs, given the axes chosen):
  $\Rightarrow P(-20^\circ, 0^\circ) + P(0^\circ, 20^\circ) \geq P(-20^\circ, 20^\circ)$
- But . . .
- Quantum theory predicts $P(\theta_1, \theta_2) = \frac{1}{2} \left( \sin \left( \frac{\mid\theta_1 - \theta_2\mid}{2} \right) \right)^2$

Results of Experiment

- Experiments have been done, and they agree with quantum mechanics every time!
- Conclusion:
  - somehow the measurement of a particle on one side affects the measurement on the other
  $\Rightarrow$ “entanglement”
Strangeness in the Quantum World?

- “entanglement”
  ⇒ “spooky” action at a distance?
- any loopholes?
- What should Christians believe?