1–2. (a) \( t = \frac{2L}{c} = \frac{2(2.74\times10^4 \text{ m})}{3.00\times10^8 \text{ m/s}} = 1.83 \times 10^{-4} \text{ s} \)

(b) From Equation 1–5 the correction \( \delta t \approx \frac{2L}{c} \times \frac{v^2}{c^2} \)

\[
\delta t = (1.83 \times 10^{-4} \text{ s})(10^{-4})^2 = 1.83 \times 10^{-12} \text{ s}
\]

(c) \( \delta c/c = \frac{4 \text{ km/s}}{299,796 \text{ km/s}} = 1.3 \times 10^{-5} \)

No, the relativistic correction of order \( 10^{-8} \) is three orders of magnitude smaller than the experimental uncertainty.

1–5. (a) In this case the situation is analogous to the boat race of Example 1-1 (and also of the Michelson-Morley experiment) with \( L = 3 \times 10^8 \text{ m}, v = 3 \times 10^4 \text{ m/s}, \) and \( c = 3 \times 10^8 \text{ m/s}. \) In the boat example, the trip across the river and back took \( 2L\gamma/c \) and the trip up and down the river took \( 2L\gamma^2/c \), where \( \gamma = 1/\sqrt{1-(v/c)^2} \), so the rower across and back wins the race. The flash going perpendicular to the motion is analogous to the boat crossing the river, and the flash forward and backward is analogous to the boat going up and down the river, with the flash in other directions taking times in between. Similarly, the fastest flashes to return will be the ones perpendicular to the motion, and the slowest those parallel with the motion, with the other directions taking times in between. If the flash occurs at \( t = 0 \), the interior is dark until about \( t = 2 \text{ s} \), at which time a bright circle of light reflected from the circumference of the great circle plane perpendicular to the direction of motion reaches the center. Then the circle splits in two, one moving toward the front and the other toward the rear, their radii decreasing to just a point when they reach the axis along the direction of motion, \( 2L\gamma(\gamma - 1)/c \approx Lv^2/c^3 = 10^{-8} \text{ s} \) after arrival of the first reflected light ring. Then the interior is again dark.

(b) In the frame of the seated observer the spherical wave expands outward at \( c \) in all directions. The interior is dark until \( t = 2 \text{ s} \), at which time the spherical wave (that reflected from the inner surface at \( t = 1 \text{ s} \)) returns to the center as a flash of light, following which the interior is dark.

1–9. Here are three different ways to look at this problem.

(a) First let’s look from the point of view of reference frame \( S \) as shown in Figure 1-14. Because this is the stationary frame with respect to the ground, the train is moving and is therefore contracted to length \( L/\gamma \). The light from the front of the train moves according to the equation \( x_f = L/2\gamma - ct \) (moving backward), and the light from the rear of the train moves according to the equation \( x_r = -L/2\gamma + ct \) (moving forward). The center of the train, which we take as the origin, moves according to \( x_c = vt \). Thus we find that when the light from the front reaches the center, \( x_f = x_c \), solving for the time elapsed we get

\[
\frac{L}{2\gamma} - ct_f = vt_f; \quad t_f = \frac{L}{2\gamma(c+v)},
\]

and from \( x_r = x_c \), when the light from the rear reaches the center, the elapsed time is

\[
\frac{-L}{2\gamma + ct_r} = vt_r; \quad t_r = \frac{L}{2\gamma(c-v)}.
\]

Thus the light from the front arrives first, and the time difference in the \( S \) frame is

\[
\Delta t = t_r - t_f = \frac{L}{2\gamma} \left( \frac{1}{c-v} - \frac{1}{c+v} \right) = \gamma \frac{v}{c^2} L.
\]

(b) In this method we solve in the frame \( S' \) of the moving train, and convert back to the \( S \) frame. The coordinates of the front (A) and the back (B) of the train at time \( t = 0 \) in the frame \( S \) are \( (x_A, ct_A) = (L/2\gamma, 0) \) and
\((x_B, ct_B) = (-L/2\gamma, 0)\) respectively. Using the transformation \(ct' = \gamma(ct - \beta x)\) from the \(S\) to the \(S'\) frame, we get

\[ct'_A = \gamma(ct_A - \beta x_A) = \gamma \left(0 - \beta \frac{L}{2\gamma}\right) = -\beta \frac{L}{2}\]

and

\[ct'_B = \gamma(ct_B - \beta x_B) = \gamma \left(0 + \beta \frac{L}{2\gamma}\right) = +\beta \frac{L}{2}\].

Hence

\[c \Delta t' = c(t'_B - t'_A) = \beta \frac{L}{2} + \beta \frac{L}{2} = \beta L,
\]

and

\[\Delta t' = \beta \frac{L}{c} = \frac{v}{c^2} L.\]

Now \(\Delta t\) appears time dilated so

\[\Delta t = \gamma \Delta t' = \gamma \frac{v}{c^2} L,
\]
as before.

(c) The third method is to use invariants. Here we note that the proper time \(c \Delta \tau\) is the same no matter what reference frame we use to calculate it. The two lightning strikes are simultaneous in \(S\) so \(t_A = t_B\), but the length of the train, which is the distance between the lightning strikes, is contracted and the appear to be at a distance \(\gamma L\) apart, so in \(S\) we calculate the proper time (squared) between two lightning strike events to be

\[c^2 \Delta \tau^2 = c^2 \Delta t^2 - \Delta x^2 = 0 - \frac{L^2}{\gamma^2} = (1 - \beta^2)L^2.\]

In \(S'\), the length of the train is just \(L\), so we have

\[c^2 \Delta \tau^2 = c^2 \Delta t'^2 - \Delta x'^2 = c^2 \Delta t'^2 - L^2.\]

Setting this equal to \(c^2 \Delta \tau^2 = (1 - \beta^2)L^2\) as obtained from frame \(S\) and solving for \(\Delta t'\), we get

\[\Delta t' = \frac{v}{c^2} L \quad \text{and therefore} \quad \Delta t = \gamma \frac{v}{c^2} L,
\]
just as before.

1–12.

(a) \(t_2 - t_1 = \gamma \left(t'_2 + \frac{vx'_2}{c^2} - t'_1 - \frac{vx'_1}{c^2}\right) = \gamma \left(t'_2 - t'_1\right)\).

(b) The quantities \(x'_1\) and \(x'_2\) in Equation 1–19 are both equal to \(x'_0\), but \(x_1\) and \(x_2\) are different and unknown.
1–15.
(a) Let frame $S$ be the rest frame of Earth and frame $S'$ be the spaceship moving at speed $v$ to the right relative to Earth. The other spaceship moving to the left relative to Earth at speed $u_x$ is the “particle”. Then $v = 0.9c$ and $u_x = -0.9c$.

\[
u_x' = \frac{u_x - v}{1 - u_x v / c^2} \quad \text{(Equation 1-22)}
\]

\[
= \frac{-0.9c - 0.9c}{1 - (-0.9c)(0.9c)/c^2} = \frac{-1.8c}{1.81} = -0.9945c.
\]

(b) Calculating as above, with $v = 3.0 \times 10^4 \text{ m/s} = -u_x$,

\[
u_x' = \frac{-3.0 \times 10^4 \text{ m/s} - 3.0 \times 10^4 \text{ m/s}}{1 - (-3.0 \times 10^4 \text{ m/s})(3.0 \times 10^4 \text{ m/s})/(3 \times 10^8 \text{ m/s})^2} = \frac{-6.0 \times 10^4 \text{ m/s}}{1 + 10^{-8}} \approx -6.0 \times 10^4 \text{ m/s}
\]

1–23.
(a)

\[L = L_p / \gamma \quad \text{(Equation 1–28)}
\]

\[= L_p \sqrt{1 - v^2 / c^2} = 1.0 \text{ m} \left[1 - (0.6c)^2 / c^2\right]^{1/2} = 0.80 \text{ m}
\]

(b)

\[t = L / v = 0.80 \text{ m} / 0.6c = 4.4 \times 10^{-9} \text{ s}.
\]

(c)

The length $\overline{OA}$ on the $x$ axis is $L$, and the length $\overline{OB}$ on the $ct$ axis yields the time $t$ that it takes for the back of the rod to reach the $x = 0$ axis. $L$ can be calculated by noting that the rod is along the $x'$ axis in the picture, and this axis is the line $ct = \beta x$. Thus because the rod is $L_p$ in length, the back end of the rod in the picture is $\beta L_p$ behind the $x = 0$ axis. So by time it gets to the $x = 0$ axis, the rod will be at $ct'' = \beta L_p$. Hence the back end of the rod will be at $(ct', x') = (\beta L_p, -L_p)$. Calculating the distance between this and the origin, we get $L = \sqrt{(x')^2 - \langle ct'\rangle^2} = \sqrt{(-L_p)^2 - (\beta L_p)^2} = L_p \sqrt{1 - \beta^2} = L_p / \gamma$. The time $t$ is of course $L / v$ as in part (b) above.
1–29.  
(a) In $S'$:  $V' = a' \times b' \times c' = (2 \text{ m})(2 \text{ m})(4 \text{ m}) = 16 \text{ m}^2$.

In $S$: Both $a'$ and $c'$ have components in the $x'$ direction.  $a' = 2 \text{ m} \sin 25^\circ = 2 \text{ m} \cdot 0.84 \text{ m} = 1.68 \text{ m}$ and $c' = 4 \text{ m} \cos 25^\circ = 3.63 \text{ m}$.

$$a_x = a' \sqrt{1 - \beta^2} = 0.84 \sqrt{1 - 0.65^2} = 0.64 \text{ m}.$$  
$$c_x = c' \sqrt{1 - \beta^2} = 3.63 \sqrt{1 - 0.65^2} = 2.76 \text{ m}.$$  
$$a_y = a' \cos 25^\circ = 2 \cos 25^\circ = 1.81 \text{ m}$$  
$$c_y = c' \sin 25^\circ = 4 \sin 25^\circ = 1.69 \text{ m}.$$  
$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(0.64)^2 + (1.81)^2} = 1.92 \text{ m}$$  
$$c = \sqrt{c_x^2 + c_y^2} = \sqrt{(2.76)^2 + (1.69)^2} = 3.24 \text{ m}.$$  

Since $b'$ is in the $z$ direction, it is unchanged, so $b = b' = 2 \text{ m}$.  

$\theta$ (between $c$ and the $x - y$ plane) = $\tan^{-1}(1.69/2.76) = 31.5^\circ$.  

$\phi$ (between $a$ and the $y - z$ plane) = $\tan^{-1}(0.64/1.81) = 19.5^\circ$.  

$V = (\text{area of } ay \text{ face}) \cdot b$ (see part (b))  

$$V = (c \times a \sin 78^\circ) \times b = (3.24 \text{ m})(1.92 \text{ m} \sin 78^\circ)(2 \text{ m}) = 12.2 \text{ m}^3.$$  

(b) 

1–33. 

$$f = \sqrt{\frac{1 - \beta}{1 + \beta}} f_0 \Rightarrow \lambda = \sqrt{\frac{1 + \beta}{1 - \beta}} \lambda_0 = \sqrt{\frac{1 + \beta}{1 - \beta}} (656.3 \text{ nm})$$

For $\beta = 10^{-3}$: $\lambda = (656.3 \text{ nm}) \sqrt{\frac{1 + 10^{-3}}{1 - 10^{-3}}} = 657.0 \text{ nm}$

For $\beta = 10^{-2}$: $\lambda = (656.3 \text{ nm}) \sqrt{\frac{1 + 10^{-2}}{1 - 10^{-2}}} = 662.9 \text{ nm}$

For $\beta = 10^{-1}$: $\lambda = (656.3 \text{ nm}) \sqrt{\frac{1 + 10^{-1}}{1 - 10^{-1}}} = 725.6 \text{ nm}$
1–43

(a) \( \gamma = \frac{1}{\sqrt{1 - \beta^2}} = \frac{1}{\sqrt{1 - (0.92)^2}} = 2.55 \)

(b) \( \tau = 2.6 \times 10^{-8} \text{ s}, \Delta t_{\text{lab}} = \gamma \tau = 2.55(2.6 \times 10^{-8} \text{ s}) = 6.63 \times 10^{-8} \text{ s} \)

(c) \( N(t) = e^{-t/\tau} \) (Equation 1-29) \( L = \sqrt{1 - \beta^2} L_0 = \sqrt{1 - (0.92)^2} (50 \text{ m}) = 19.6 \text{ m} \) where \( L \) is the distance travelled in the pion system. The time they cover this distance is \( t = x/v = x/\beta c = 19.6 \text{ m} / (0.92 \times 3 \times 10^8 \text{ m/s}) = 7.10 \times 10^{-7} \text{ s} \). So for \( N_0 = 50,000 \) pions initially, at the end of 50 m in the lab,

\[ N = (5.0 \times 10^4) e^{-7.1/2.6} = 3,260. \]

(d) Ignoring relativity, the time to cover 50 m at 0.92c is 1.81 \times 10^{-7} \text{ s} and \( N \) would then be

\[ N = (5.0 \times 10^4) e^{-18.1/2.6} = 47. \]

1–45

(a)

(b) Note that \( x' = \gamma(x - \beta ct) \), so the \( ct' \) axis is the line for which \( x' = 0 \), which is given by \( ct = (1/\beta)x \). Since the explosions occur at the same place in \( S' \), in \( S \) they occur along a line parallel to the \( ct' \) axis. At \( t = 0 \), \( x = 480 \text{ m} \), and at \( t = 5 \mu\text{s} \), \( x = 1200 \text{ m} \), so the slope of the \( ct' \) axis in the \( ct-x \) plane is

\[ \frac{c \Delta t}{\Delta x} = 3 \times 10^8 \text{ m/s} \frac{5 \mu\text{s}}{(1200 \text{ m} - 480 \text{ m})} = 2.08 = \frac{1}{\beta} = \frac{c}{v}. \]

So \( \beta = 1/2.08 = 0.48 \) and \( v = 0.48c = 1.44 \times 10^8 \text{ m/s} \).

(c) Since the events happened in \( S' \) at the same location, \( S' \) measured the proper time. \( \gamma \) for the situation is 1.14, so the elapsed time interval in \( S' \) is 5 \( \mu\text{s} / \gamma = 4.39 \mu\text{s} \), or the distance along the \( ct' \) axis will be 1316 m.

(d) The proper time \( \Delta \tau \) between the two explosions, which is the “distance” between the two events (in a space-time sense), can be calculated in any reference frame. Since the explosions take place at the same location in the frame \( S' \), the proper time is just the regular time \( \Delta \tau = \Delta t' \). In the frame \( S \), the proper time is given by

\[ c \Delta \tau = \sqrt{c^2 \Delta t'^2 - \Delta x^2} = \sqrt{(3 \times 10^8 \text{ m/s})^2 (5 \times 10^{-6} \text{ s})^2 - (720 \text{ m})^2} = 1316 \text{ m}, \]

so

\[ \Delta t' = \frac{1316 \text{ m}}{c} = 4.39 \times 10^{-6} \text{ s} = 4.39 \mu\text{s}. \]

We could of course also use time dilation \( \Delta t = \gamma \Delta t' \) to arrive at the same conclusion.